

$$\text{first Order} = f_0 + (x_{\text{new}} - x_0) * f'_1$$

$$\text{Second Order} = f_0 + (x_{\text{new}} - x_0) * f'_1 + (x_{\text{new}} - x_0) * (x_{\text{new}} - x_1) * f''_2$$

$$\text{Third Order} = f_0 + (x_{\text{new}} - x_0) * f'_1 + (x_{\text{new}} - x_0) * (x_{\text{new}} - x_1) * f''_2 \\ + (x_{\text{new}} - x_0) * (x_{\text{new}} - x_1) * (x_{\text{new}} - x_2) * f'''_3$$

$$f_0 = f_{x_0}$$

$$\underline{f_1} = F[x_1, x_0] = \frac{f_{x_1} - f_{x_0}}{(x_1 - x_0)}$$

$$\underline{f_2} = F[x_2, x_1, x_0] = \frac{F[x_2, x_1] - F[x_1, x_0]}{(x_2 - x_0)}$$

$$= \frac{\frac{f_{x_2} - f_{x_1}}{x_2 - x_1} - f_1}{(x_2 - x_0)}$$

$$\underline{f_3} = F[x_3, x_2, x_1, x_0] = \frac{F[x_3, x_2, x_1] - F[x_2, x_1, x_0]}{x_3 - x_0}$$

$$= \frac{\frac{\frac{f_{x_3} - f_{x_2}}{x_3 - x_2} - \frac{f_{x_2} - f_{x_1}}{x_2 - x_1}}{x_3 - x_1} - f_2}{x_3 - x_0}$$