EE 202 - Mathematical Techniques in Electrical Engineering

LAB 8

Question:

Solve the following problem by using FEM;

$$-\frac{d^2u}{dx^2} - u = -x^2, \quad 0 < x < 1, \quad u(0) = 0, \quad u(1) = 0$$

Exact solution is as follows;

$$u_{exact} = \frac{\sin(x) + 2\sin(1-x)}{\sin(1)} + x^2 - 2$$

**Find the matrices by hand calculation, then solve the system by using Matlab.

Solution:

```
%First part
syms x u1 u2 u3 u4 u5 Phi I;
% Phi(i,j) is the i-th approx. funct. over element j
Phi(1,1) = 1-4*x;
Phi(2,1) = 4*x; Phi(2,2) = 2-4*x;
Phi(3,2) = 4*x-1; Phi(3,3) = 3-4*x;
Phi(4,3) = 4*x-2; Phi(4,4) = 4-4*x;
Phi(5,4) = 4*x-3;
coord = [0, 0.25, 0.5, 0.75, 1];
for i=1:5 % Integral loop
 I(i) = 0; % Initialize the i-th integral to zero.
 for e=1:4 % Element loop
 w = Phi(i,e);
 dwdx = diff(w, x);

 u = u1*Phi(1,e) + u2*Phi(2,e) + u3*Phi(3,e) + u4*Phi(4,e) + u5*Phi(5,e);
 dudx = diff(u, x);

 I(i) = I(i) + int(dwdx*dudx - w*u + w*x^2, x, coord(e), coord(e+1));
 end
end

%Second part
coord = [0 0.25 0.5 0.75 1];
u = [0 -0.0232 -0.0405 -0.0392 0];
plot(coord, u, 'o-b', 'LineWidth', 2)
hold
x = 0:0.01:1;
uExact = (sin(x) + 2*sin(1-x)) / sin(1) + x.*x - 2;
plot(x, uExact, 'r', 'LineWidth', 2)
grid on
```